Solutions

Exam 3
Chapters 3,4 and 5

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Give exact answers. Indicate your final answer with a $\boxed{\text{box}}$.

1. (8 points) Evaluate the following logarithms without using a calculator. (Show at least one step of work for credit.)

- $\log_3(\frac{1}{27})$
- $\log_8(2)$

$$\log_3(1/27) = \log_3(1/3^3) = \log_3(3^{-3}) = -3$$

$$\log_8(2) = \log_8(8^{1/3}) = 1/3$$

---

2. (6 points) Use logarithm rules to combine the following into a single logarithm.

$$2\ln(a+b) + 2\ln(a-b) - \ln(c).$$

$$\parallel$$

$$\ln((a+b)^2) + \ln((a-b)^2) - \ln(c)$$

$$\parallel$$

$$\ln\left(\frac{(a+b)^2(a-b)^2}{c}\right) = \ln\left(\frac{(a^2-b^2)^2}{c}\right).$$

3. A patient is administered 180 mg of a therapeutic drug. It is known that 30% of the drug is expelled every hour.

(a) (2 points) Find an exponential model for the amount of drug remaining in the patient's body after t hours.

(b) (3 points) Use the model to predict the amount of the drug that remains in the patient's body after 6 hours.

(c) (3 points) Use the model to predict how long it will take before there is only 30 mg of the drug remaining in the patient's body.

(a) $f(t) = 180(0.7)^t$

(b) $f(6) = 180(0.7)^6 \approx 21.18$ mg

(c) $30 = 180(0.7)^t$

$$\frac{1}{6} = (0.7)^t$$

$$\log(1/6) = t\log(0.7)$$

$$t = \frac{\log(1/6)}{\log(0.7)} \approx 5.02 \text{ hours}$$

4. A bacterial infection starts with 1500 bacteria and the bacterial count quadruples every 8 hours.

(a) (3 points) Find an exponential growth model for the number of bacteria after x 8 hour time periods.

(b) (3 points) Find an exponential growth model for the number of bacteria after t hours.

(a) $f(x) = 1500 \cdot 4^x$

(b) $f(t) = 1500 \cdot 4^{t/8} = 1500 \cdot (1.189)^t$

5. Shalan invests \$5000 dollars into investment option A that earns 6% interest each year, compounded semiannually. He also invests \$3000 dollars into investment option B that earns 9% interest each year, compounded continuously.

(a) (3 points) Find a model for the amount of money accrued in investment A after t years.

(b) (3 points) Find a model for the amount of money accrued in investment B after t years.

(c) (2 points) How many years will it take before investment B outgrows investment A?

(a) $f(t) = 5000\left(1+\frac{.06}{2}\right)^{2t} = 5000(1.03)^{2t}$

(b) $g(t) = 3000\, e^{0.09t}$

(c) $g(t) = f(t)$

$3000\, e^{0.09t} = 5000\,(1.03)^{2t}$

$\ln(3000 e^{0.09t}) = \ln(5000 (1.03)^{2t})$

$\ln(3000) + 0.09t = \ln(5000) + 2t\ln(1.03)$

$0.09t - 2t\ln(1.03) = \ln(5000) - \ln(3000)$

$t(0.09 - 2\ln(1.03)) = \ln(5000) - \ln(3000)$

$t = \dfrac{\ln(5000) - \ln(3000)}{0.09 - 2\ln(1.03)} \approx 16.54$ years

6. (4 points) Determine if the two functions below are inverses of each other.

$$h(x) = 10 \cdot 4^x \quad \text{and} \quad l(x) = \log_4(\frac{x}{10}).$$

$h(l(x)) = h(\log_4(\frac{x}{10})) = 10 \cdot 4^{\log_4(\frac{x}{10})} = 10 \cdot \frac{x}{10} = x$ ✓

$l(h(x)) = l(10 \cdot 4^x) = \log_4(10 \cdot \frac{4^x}{10}) = \log_4(4^x) = x$ ✓

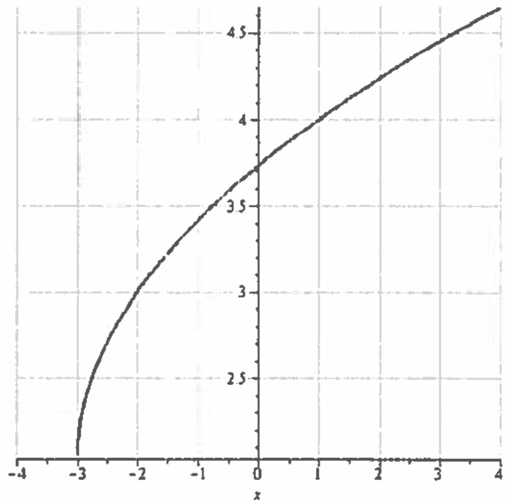
Yes, they are inverses.

---

7. (6 points) Let $f(x) = 2x^2$ and $g(x) = x - 1$. Find $f(g(x))$.

$f(g(x)) = f(x-1) = 2(x-1)^2 = 2x^2 - 4x + 2$

8. (4 points) Consider the function given by the graph below. Is it invertible? Explain your reasoning.



Yes, it passes the horizontal line test.